DO NOW

$\sin^2 x + \cos^2 x =$ 1

$1 + \tan^2 x =$ $\sec^2 x$

$1 + \cot^2 x =$ $\csc^2 x$

3.3 Product and Quotient Rules - Day 4

Derivatives of Trigonometric Functions

Recall:

$\frac{d}{dx}[\sin x] =$ $\cos x$ $\frac{d}{dx}[\cos x] =$ $-\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

Prove: $\frac{d}{dx}[\tan x] = \sec^2 x$

$$\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x}$$

$$\left(\frac{1}{\cos x}\right)^2$$

$$\sec^2 x$$

Examples: Find the derivatives.

1. $y = 2\sin x - \tan x$

$y' = 2\cos x - \sec^2 x$

2. $g(t) = 2\sec t + 3\tan t$

$g'(t) = 2\sec t \tan t + 3\sec^2 t$

$g'(t) = \sec t(2\tan t + 3\sec t)$

3. $y = x\tan x$

$y' = x\sec^2 x + \tan x$

4. $h(t) = e^t \csc t$

$h'(t) = e^t(\csc t \cot t) + \csc t(e^t)$

$h'(t) = e^t \csc t(-\cot t + 1)$

$h'(t) = e^t \csc t(1 - \cot t)$

5. $f(x) = \frac{\tan x}{1 - 2x}$

$$f'(x) = \frac{(1-2x)(\sec^2 x) - \tan x(-2)}{(1-2x)^2}$$

$$f'(x) = \frac{\sec^2 x - 2x\sec^2 x + 2\tan x}{(1-2x)^2}$$

Higher Order Derivatives:

s(t) → position function

s'(t) = v(t) → velocity function

s''(t) = v'(t) = a(t) → acceleration function

* rate at which the velocity is changing with respect to time (what you feel when you step on the gas pedal.)

**Taking derivatives of derivatives of functions.

Notation:

First derivative: $f'(x)$, y', $\frac{dy}{dx}$, $\frac{d}{dx}[f(x)]$

Second derivative: $f''(x)$, y'', $\frac{d^2y}{dx^2}$ $\frac{d^2}{dx^2}[f(x)]$

Third Derivative: $f'''(x)$, y''', $\frac{d^3y}{dx^3}$, $\frac{d^3}{dx^3}[f(x)]$

Fourth derivative: $f^{(4)}(x)$, $y^{(4)}$, $\frac{d^4y}{dx^4}$, $\frac{d^4}{dx^4}[f(x)]$

⋮

nth derivative: $f^{(n)}(x)$, $y^{(n)}$ $\frac{d^ny}{dx^n}$, $\frac{d^n}{dx^n}[f(x)]$

Ex: $y = x^4 - 3x^3 + 2x^2 - \sin x$

$y' = 4x^3 - 9x^2 + 4x - \cos x$

$y'' = 12x^2 - 18x + 4 + \sin x$

$y''' = 24x - 18 + \cos x$

HOMEWORK

pg 147; 41 - 58

pg 147 - 149; 63, 65, 67 - 71 odd (part a only),
73 - 79 odd, 83, 91, 93, 97,
99, 101, 105 - 108